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RICHARD T. ELY LECTURE

THE ECONOMICS OF PRODUCTION

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For the last twenty years or so I have singled myself out among my fellow econometricians for arguing with all the means at my disposal that not every element of the economic process can be related to a number and, consequently, that this process cannot be represented in its entirety by an arithmomorphic model. At the same time, I have insisted that in our haste to mathematize economics we have often been carried away by mathematical formalism to the point of disregarding a basic requirement of science; namely, to have as clear an idea as possible about what corresponds in actuality to every piece of our symbolism. Curiously, in the home of quantity, in the natural sciences, this position does not constitute a singularity. On the contrary, essentially the same words of caution have come from many a high authority in physics—such as Max Planck or Percy William Bridgman, for example.[1] But even some engineers have raised their voices against blind symbolism. The recent remarks by a well-known British engineer are worth quoting at length:

> Contrary to common belief it is sometimes easier to talk in mathematics than to talk in English; this is the reason why many scientific papers contain more mathematics than is either necessary or desirable. Contrary to common belief it is also often less precise to do so. For mathematical symbols have a tendency to conceal the physical meaning that they are intended to represent; they sometimes serve as a substitute for the arduous task of deciding what is and what is not relevant; . . . . It is true that mathematics cannot lie. But it can mislead.
>
> However, the dangers of over-indulgence in formula spinning are avoided if mathematics is treated, wherever possible, as a language into which *thoughts may only be translated after they have first been* [*clearly*] *expressed in the language of words*. The use of mathematics in this way is indeed disciplinary, helpful, and sometimes indispensable.[2]

The topic of this lecture—the economics of production—presents, I believe, sufficient interest by itself. But in choosing it, I have been guided also by the fact that it may serve as a substantial illustration of the harm caused by the blind symbolism that generally characterizes a hasty mathematization.

I

What has come to be known as "the production function" is quite an old item in the economist's analytical paraphernalia. As we may recall, it was introduced in 1894 by Wicksteed with one simple remark: "*the product being a function of the factors of production we have* $P=f(a, b, c, \ldots)$."[3] This paradigm of imprecision apparently sufficed to make us accept Wicksteed's simple symbolism as an adequate analytical representation of any production process and use it indiscriminately in every kind of situation. And as the usage of the vapid terms "input" and "output" became widespread, popular manuals came to treat the subject in an even more cavalier manner than Wicksteed's. A typical presentation is that the production function expresses symbolically the fact that "the output of the firm depends on its inputs."

But even consummate economists have accepted Wicksteed's formula without any ado. They only felt that the meaning of the variables involved ought to be explained. The greater number of such authors adopt the position that the formula shows the quantities of inputs (or of factors) necessary to produce a certain quantity of output (or of product). Accordingly, all symbols in a production function,

$$Q = F(X, Y, Z, \cdots), \tag{1}$$

stand for quantities.[4] Others conceive the same function as a relation between the inputs per unit

[1] Max Planck, *The New Science* (New York, 1959), pp. 43, 158–59; P. W. Bridgman, *The Logic of Modern Physics* (New York, 1949), p. 50.

[2] Reginald O. Kapp, *Towards a Unified Cosmology* (New York, 1960), p. 111. My italics.

[3] Philip H. Wicksteed, *The Co-ordination of the Laws of Distribution* (London, 1894), p. 4.

[4] For a small yet representative sample, see A. L. Bowley, *The Mathematical Groundwork of Economics* (Oxford, 1924), pp. 28–29; J. R. Hicks, *The Theory of Wages* (London, 1932), p. 237; E. Schneider, *Theorie der Produktion* (Vienna, 1934), p. 1; A. C. Pigou, *The Economics of Stationary States* (London, 1935), p. 142; P. A. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass., 1948), pp. 57–58; K. E. Boulding, *Economic Analysis* (3rd ed., New York, 1955), p. 585; Sune Carlson, *A Study on the Pure Theory of Production* (New York, 1956), p. 12; Ragnar Frisch, *Theory of Production* (Chicago, 1965), p. 41.

of time and the output per unit of time; i.e., as a relation

$$q = f(x, y, z, \cdots), \tag{2}$$

in which all symbols stand for rates of flow.[5]

Curiously, no one seems to have been intrigued by the existence of these entirely distinct interpretations. Instead, many economists—including some analytical authorities—have used both definitions indifferently, sometimse even on the same page.[6] The undeniable inference is that the economic profession considers relations (1) and (2) as two completely equivalent ways of representing any production process whatsoever. Yet behind this belief there lies an analytical imbroglio which is easily brought to light.

We need only recall that the production function is a tool associated with a static process or, to use a more explicit expression, with a steady-going process. For such a process, the following relations

$$Q = tq, \; X = tx, \; Y = ty, \cdots \tag{3}$$

hold for any time interval t and for the quantities of product and of factors corresponding to that interval. With the aid of these relations and (2), relation (1) becomes

$$tf(x, y, z, \cdots) = F(tx, ty, tz, \cdots). \tag{4}$$

And since this relation must be true for any t, it follows, first, that f and F are one and the same function,

$$f(x, y, z, \cdots) \equiv F(x, y, z, \cdots), \tag{5}$$

and, second, that this function is homogeneous of the first degree. Therefore, the tacit presumption that the forms (1) and (2) are equivalent implies that the returns to scale must be constant in absolutely every production process.

Nothing, I believe, need be added to convince ourselves that this imbroglio is the direct consequence of our acceptance of Wicksteed's symbolism without first probing its validity as an analytical mirror of actuality. This conclusion raises a new and troublesome issue. Does either of the forms, (1) or (2), constitute an adequate representation of a process of production and, if so, what kind of process may be represented by it? For a start, let us try to examine it in its broad lines.

[5] G. Stigler, *The Theory of Competitive Price* (New York, 1942), p. 109; T. C. Koopmans, "Analysis of Production as an Efficient Combination of Activities," in *Activity Analysis of Production and Allocation*, ed. T. C. Koopmans (New York, 1951), p. 35.

[6] E.g., Frisch, *op. cit.*, p. 43.

II

Before anything else, we should note that for no other branch of economics is the concept of process as essential as for the economics of production. But, widely used though the word "process" is in sciences and philosophy, the literature seems to offer no specific definition of it. Now and then, the concept is merely associated with change. However, change is a notoriously intricate notion which has kept philosophers divided into two opposing camps: one maintaining that all is only being; the other, that all is only becoming. Obviously, science can follow neither of these teachings. Nor can it, unfortunately, embrace Hegel's dialectical synthesis that being is becoming. Analytical science must distinguish between object and event. Consequently, it must embrace the so-called "vulgar" philosophy—according to which there are both being and becoming—and cling to it to the very end. The upshot is that science must find a way to represent a process analytically.

It is obvious that, for this purpose, we must retain one point of dialectics; namely, that change and, hence, process cannot be conceived otherwise than as a relation between some entity and its complement in the absolute whole. In viewing a living tree as a process we oppose that tree to everything else—to "its other," in Hegel's terminology. Only for the absolute whole—the universe in its eternity—has change no meaning: such a totality has no complement. The notion of partial process, therefore, implies some slits cut into the absolute whole. But as a long series of thinkers, from the ancient Anaxagoras to the modern Niels Bohr, have taught us, this whole is seamless.[7] However, in this case as in all others, analysis must proceed by some heroic simplifications and totally ignore their ultimate consequences.

The first heroic step is to divide actuality into two parts—one representing the partial process in point; the other, its environment (so to speak)—separated by a boundary consisting of an arithmomorphic void. For if the boundary would not be such a void, we would get three parts instead of two and, as is easily seen, we would be drawn into a dialectical infinite regress. So, all that exists in actuality at any moment must belong either to a process or to its environment. The basic element of the analytical picture of a process is, therefore, the boundary. No analytical boundary, no analytical process.

[7] See Fragment 8 in J. Burnet, *Early Greek Philosophy* (4th ed., London, 1930), p. 47; Niels Bohr, *Atomic Physics and Human Knowledge* (New York, 1958), p. 10.

Now, precisely because actuality is a seamless whole we can slice it wherever we may please. And, Plato to the contrary,[8] actuality has no joints to guide a carver. As economists we know only too well the unsettled issue of where the natural boundary of the economic process lies. Only our particular purpose in each case can guide us in drawing the boundary of a process. So, every scientist slices actuality in the way that suits best his own objective—an operation that cannot be performed without some intimate knowledge of the corresponding phenomenal domain.

An analytical boundary, as conceived here, must consist of two components. Like a frontier, one component separates the process at any time from the rest of actuality, although we must not think that this frontier is necessarily geographical in nature or rigidly determined. Witness the process of thought itself or that of an acorn growing into an oak. The second component is the duration of the process, determined by the time moments at which the process we have in mind begins and ends. Naturally, these moments must be at a finite distance; otherwise, we would not know all that has gone into the process or all that the process does. Nor must we allow them to coincide. For, to recall Whitehead's admonition,[9] a durationless process, an event at an instant of time as a primary fact of nature, is nonsense.[10] We can then choose the time scale so that the process begins at $t=0$ and ends at $t=T$, with $T>0$. T is the duration of the process, but for a production process we may prefer, instead, Marx's term: the time of production.

The next point is truly crucial: in saying that a given analytical process begins at $t=0$ and ends at $t=T$ we must take the underscored words in their strictest sense. Before $t=0$ and after $t=T$ the analytical process is out of existence. That is, in conceiving such a process we must totally abstract from it what happens outside the duration we have assigned to the process. The mental operation is clear: an analytical process must be viewed as a hyphen between one *tabula rasa* and another.

Our next problem is to arrive at an analytical description of the happening, associated with a given process. Because of the principle, "No analytical boundary, no analytical process," analysis

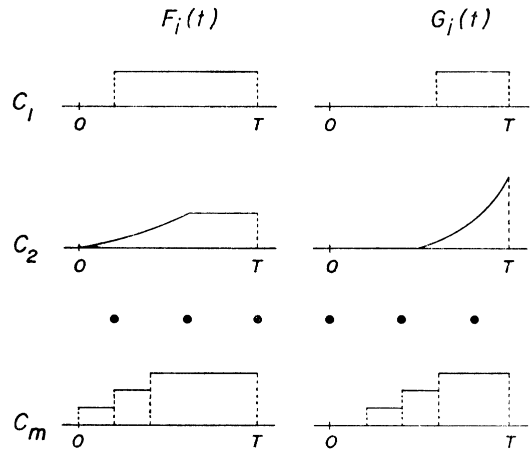

FIGURE 1

must renounce any hope of including in this description the happenings inside an analytical process. Indeed, in order to describe analytically what happens inside a process, we must divide it by a new boundary into two new processes to which the same rule will apply. And so on ad infinitum. The analytical description of a process, therefore, reduces to recording everything that crosses the boundary in either direction. In connection with this picture we can endow the terms input and output with quite precise meanings.

Analysis now needs to take another heroic step —to assume that the number of elements involved is finite and that every element is cardinally measurable (which implies that every element is a homogeneous entity). If $C_1, C_2, \ldots, C_m$ denote the distinct elements, the analytical description is complete if for every C_i we have determined two nondecreasing functions $F_i(t)$ and $G_i(t)$ over the closed interval $[0, T]$, the first function showing the cumulative input, the second the cumulative output up to time t (inclusive). Any analytical process—whether in economics or any other domain—may therefore be represented graphically by a series of curves, as in Figure 1.

In a plastic image, the coordinates of an analytical process may be likened to continuously reported data of import and export, with one important detail. Since in describing a process analytically we must begin and end with a *tabula rasa*, this hall (in which we are now gathered) must be listed both as input and as output in the process consisting of the delivery of this lecture. In the analytical approach we are not interested in how this hall came into being or in its use before or after this lecture. However, we must recognize

[8] Plato, *Phaedrus*, p. 265.

[9] Alfred North Whitehead, *An Enquiry Concerning the Principles of Natural Knowledge* (2nd ed., Cambridge, England, 1925), p. 2; also his *The Concept of Nature* (Cambridge, England, 1930), p. 57.

[10] All this does not mean that, in the next stage of our inquiry, we cannot arrive at the excluded cases by a passage to the limit.

that, as the result of every use, the hall suffers some wear and tear, imperceptible though this may be. Similarly, in any production process the same person must be listed as a rested worker among inputs and as a tired worker among outputs. A tool, too, may go in new and come out used. But even though we recognize the rested and the tired worker as being the same man, we must treat the former as a different element from the latter. Each element of an analytical process—as we have decided—must be completely homogeneous, a condition that does not always cover sameness.

These cases, of the worker and of the tool, raise a troublesome problem for the economist. The reason is that our material of study is the commodity. We slice the economic domain into units of production and units of consumption because at the boundaries thus drawn we can catch every commodity. Drawing a boundary in a glass plant between the melted glass furnace and the rolling machines would serve none of the economist's purposes: at this time, melted glass is not a commodity.[11] Briefly, the economist cannot afford to abandon his commodity fetishism any more than the chemist, for example, can renounce his fetishism of the molecule.

The difficulty, which at bottom is related to qualitative change, is that even though we cannot avoid including "tired worker" and "used tool" in the list of outputs of any production process, neither category fits the usual notion of commodity. Our entire analytical edifice would collapse if we were to accept the alternative position that the aim of economic production is to produce not only the usual products but also tired workers and used tools.

A new heroic step is needed to eliminate this difficulty. It consists of the familiar, old fiction of a process in which capital is maintained constant. The fiction does raise some analytical issues, for if all tools and all workers are to be maintained at a constant level of efficiency, any production process will have to include most of the enterprises and households in the world. Factually, however, the fiction is not more, not less reasonable than that of frictionless movement in mechanics. A simple glance at the activity inside a plant or a household suffices to convince us that efforts are constantly directed not toward keeping durable goods physically self-identical (which is quite impossible), but toward maintaining them in good working condition. And this is all that counts in production. The only factor we need neglect is the daily wear and tear of the worker. This is not too much to demand, since the worker is daily restored in the household.

The elements may now be divided into two relevant categories. In the first category we shall place those elements that appear as input and as output and are related by reason of sameness or of equality of quantity. A piece of Ricardian land, a motor, the amount of clover used as seed in growing clover seed (not clover fodder!), or a worker, illustrate this category. To elements such as these I propose to refer as funds so as to emphasize their economic invariableness. The other elements, which appear only as input or only as output, constitute the category of flows. Obviously, since the fund elements are maintained, the process may be activated again provided that the necessary inflows are still forthcoming. Labeled variously as a static or as a stationary process, or, still, as a diagram of simple reproduction, this fiction constitutes the fundamental element in the analysis of production from the classical to the hypermodern school. Reproducible, however, seems to describe the process better. The analytical picture we have thus reached is worth stressing: in a reproducible process, the fund elements are the immutable agents that transform some input flows into output flows. No picture of a process—whether static or dynamic—is complete if it does not include both categories of elements.[12] And the essential difference between these categories calls for a different representation of the fund coordinates. A flow coordinate will continue to be represented, according to the case, by the cumulative input or the cumulative output; i.e., by a quantity of some substance. Because in case of a fund the input and the output are economically the same substance, the coordinate of a fund may be represented by the difference $F_i(t)-G_i(t)$. But to maintain a convenient symmetry with the flow coordinates, we may use instead the cumulative amount of that intangible entity usually called the service of the fund.

III

In the case of a production process, the elements may be classified into some fruitful categories. The inflows that are transformed by the agents may come either from nature or from other

[11] That technological innovations may change this situation is evidenced by ready-mix cement and brown-and-serve rolls, for example, which only recently have become commodities.

[12] A point on which I insisted long ago: cf. my article "Aggregate Linear Production Function and Its Applications to von Neumann's Model" in *Activity Analysis of Production and Allocation*, ed. T. C. Koopmans (New York, 1951), pp. 100–01.

production processes; we shall denote them generically by (R) and (I). There also are inflows, (M), earmarked for maintenance. The output flows consist of products, (Q), and waste, (W). Finally, the funds include Ricardian land (L), capital equipment (K), and—to use Marx's very appropriate term—labor power, (H). With these notations, the analytical representation of a reproducible process is

$$(6)\quad [\underset{0}{\overset{T}{R}}(t), \underset{0}{\overset{T}{I}}(t), \underset{0}{\overset{T}{M}}(t), \underset{0}{\overset{T}{Q}}(t), \underset{0}{\overset{T}{W}}(t); \underset{0}{\overset{T}{L}}(t), \underset{0}{\overset{T}{K}}(t), \underset{0}{\overset{T}{H}}(t)];$$

that is, a set of functions, which defines a point in an abstract (functional) space.

This is a far cry from the notion inherited from Wicksteed, according to which a process is represented by a point in the ordinary (Euclidean) space. The superiority of (6) over the point representation needs no elaborate argument. In (6) we have a complete set of instructions on how to set up the corresponding process. The form also reminds us continuously that a process has a duration, a time of production. Nothing is missing from it.

The difference yields an entirely new form for the production function. Since by a production function we must still understand the set of all processes that transform the same input flows into an outflow of the same product, from (6) it follows that the production function must be a relation among functions, instead of numbers. This relation, which may be written after the old pattern as

$$(7)\quad \underset{0}{\overset{T}{Q}}(t) = \mathfrak{F}[\underset{0}{\overset{T}{R}}(t), \underset{0}{\overset{T}{I}}(t), \underset{0}{\overset{T}{M}}(t), \underset{0}{\overset{T}{W}}(t); \underset{0}{\overset{T}{L}}(t), \underset{0}{\overset{T}{K}}(t), \underset{0}{\overset{T}{H}}(t)],$$

is what the mathematicians call a functional.

The results just reached call for numerous observations. Here, I can take up only a few and touch upon them briefly.[13]

First, the reason why I have excluded no element from the categories listed in (6), is that the scholar must never prejudge. Even an economist must first arrive at a complete description of a process and only then decide which elements may be left out because they are economically irrelevant. Nature does not indeed have a cashier's window where we may pay her for the elements (R); yet it would be utterly inept to ignore in the economics of production the fact that natural resources are neither inexhaustible nor uniformly distributed over the globe. The type of economic model now in vogue, which assumes that growth normally proceeds at a constant rate, simply blots out the most numerous and most critical cases—such as Somaliland or our own Appalachia, for instance. One may feel even more tempted to leave out the waste category, on the ground that waste by definition has no value. But again, as we have come to recognize recently on an increasing scale, the existence of waste is not an innocuous aspect of the economic process.

Second, we should not fail to observe that, since a fund enters a reproducible process and comes out without any impairment of its economic efficiency, service is the only way by which it can participate in the production of the product. While it is true that the cloth—an inflow element—effectively passes into the coat, the same cannot apply to the needle—a fund element. And if one finds the needle in the coat just bought, it certainly is a regrettable accident. The point is that the problem of how the contribution of a fund affects the value of the product is not as directly simple as in the case of a flow factor.

Third, both the value of a fund's service and that of its maintenance flow must, in principle, be imputed to the value of the product. Contrary to Marx's teachings—which have gradually infiltrated the thinking of many a standard economist—there is no economic double counting in this. No worker, no lecturer, can discharge his duties by sending to the shop or to the classroom only that "definite quantity of muscle, nerve, brain, etc., [which] is wasted" during work—as Marx claimed.[14] When one works, one must be present with his entire fund of mental and physical capabilities. The same is true for all other funds: the bridge must be there in its full material existence before we can cross the river. If it were true that we could cross a river on the maintenance flow of a bridge or drive the maintenance flow of an automobile on the maintenance flow of a highway, there would be little financial difficulty in saturating the whole world with all the river crossings and automotive facilities. Economic development could be brought about everywhere with practically no waiting.

IV

As with almost everything else, among the various processes we may envisage in production there is one process that fits the epithet "elementary." It is the process by which every unit of the product—a single piece of furniture or a molecule of gasoline—is produced. The process is

[13] For greater details see Chapter IX in my forthcoming volume, *Entropy and the Economic Process* (Harvard Univ. Press).

[14] Karl Marx, *Capital* (Chicago, 1932), Vol. I, p. 190.

directly observable in the shop of a cabinetmaker, but it can be easily determined even in a large-scale enterprise. Whatever the product, one thing is certain about the elementary process. In relation to it, most of the funds are idle over large periods of time. The plow is needed only a few days during the whole production time of growing a corn plant; the same is true for the saw or the plane in the production of a table. There is no exception to this rule. And, I contend, one of the most important aspects of the economics of production is how to minimize these periods of fund idleness, whether we are thinking of man, capital equipment, or land.[15]

Now, if only one table is demanded during the time of production, T, then obviously we need operate only one elementary process after another in succession. But if n tables, with $n>1$, are demanded during T, then two alternatives are open to us. We may start n processes at the same time and repeat the operation when they are ended. This is the arrangement in parallel. The second arrangement is the arrangement in line, in which equal batches of processes are begun one after another at intervals equal to an aliquot part of T.

It is obvious that the production function of a system in which the elementary processes are arranged in parallel is obtained from (7) by multiplying every coordinate by n:

$$(8)\quad [n\overset{T}{\underset{0}{Q}}(t)]=\mathfrak{F}\{[n\overset{T}{\underset{0}{R}}(t)], \cdots, [n\overset{T}{\underset{0}{W}}(t)]; [n\overset{T}{\underset{0}{L}}(t)], \cdots\}.$$

The point that deserves to be stressed is that the arrangement in parallel offers little or no economic gain. Most kinds of fund factors are now needed in an amount n times as great as in the elementary process. In addition, the idleness of each such fund factor is *ipso facto* amplified by n. The only exceptions are the fund factors that—like a large bread oven, for instance—may accommodate several elementary processes simultaneously. But even though the capacity of such a fund factor would be more fully utilized, its idleness period would remain the same.

The situation completely changes for the arrangement in line. If we assume away any incommensurabilities among the time periods involved in the schedule of the elementary process—an inevitable assumption in practice—and if n is sufficiently large, then a number of processes can be arranged in line so that no fund is idle at any time.[16] The situation is vividly exemplified by an assembly line, in which every tool and every worker shift without interruption from one elementary process to the next. The arrangement in line, however, describes any factory. In a factory, therefore, the economy of time reaches its maximum. This conclusion opens an avenue of utmost importance. To explore it, we may begin by determining the analytical representation of a factory process.

In a first approach we may consider the entire physical plant as one monolithic fund, P. Over an arbitrary interval $[0, t]$, during which the factory process is in operation, the coordinate of this fund is the function Pt. Similarly, the coordinate of labor power is Ht. And if for the convenience of diction we assume that all flow elements are continuous, their coordinates, too, are represented by linear homogeneous functions. Thus (7) becomes:

$$(9)\quad (\overset{t}{\underset{0}{qt}}) = \mathcal{G}[(\overset{t}{\underset{0}{rt}}), (\overset{t}{\underset{0}{it}}), (\overset{t}{\underset{0}{mt}}), (\overset{t}{\underset{0}{wt}}); (\overset{t}{\underset{0}{Pt}}), (\overset{t}{\underset{0}{Ht}})].$$

Let us note that this is a very special functional: first, every function involved in it depends upon a single parameter and, second, the value of t is entirely arbitrary. For these reasons, the functional degenerates into a point function.

There are two degenerate forms. The first is

$$(10)\quad q = \Theta(r, i, m, w; P, H).$$

This formula reminds us of one of the current interpretations mentioned in Section I; namely, that of relation (2). We should note, however, that θ involves two dimensionally different categories of variables. The lower case symbols represent flow rates of some substances. The upper case symbols stand for the rates of service per unit of time. Strangely, however, these last rates do not involve the time element at all: P stands for the plant, and H, for the total labor power—briefly, for quantities of some substances. The second degenerate form is

$$(11)\quad Q = \Psi(R, I, M, W; \mathcal{P}, \mathcal{H}; t).$$

Here, the symbols in roman capital letters are again quantities of some substances; those in script letters are services, and t is the period with which these quantities and services are associated. The form (11), in turn, reminds us of relation (1); i.e., of the quantity interpretation of Wicksteed's formula. The most important difference is that Ψ includes time as an explicit variable.

[15] A period of idleness is characterized by the constancy of the corresponding fund coordinate.

[16] The number of elementary processes that should be started each time is the smallest common multiple of the numbers of such processes that can be accommodated at the same time by each unit of the various funds. Batches should be started at intervals of T/d, d being the greatest common divisor of T and of the intervals during which the various funds are needed in an elementary process.

This difference bears upon the earlier argument that Wicksteed's production function is homogeneous of the first degree. Actually, Ψ is such a function—as easily follows from the identities

$$(12) \quad Q = qt, \cdots, W = wt, \mathcal{P} = Pt, \mathcal{H} = Ht,$$

analogous to (3). There is then an intimate relation between (10) and (11); namely,

$$(13) \quad t\Theta(r, i, m, w; P, H) = \Psi(R, I, M, W; \mathcal{P}, \mathcal{H}; t).$$

Hence,

$$(14) \qquad \Theta = [\Psi]_{t=1}.$$

The imbroglio created by (5) is thus resolved. Of course, this does not mean that the factory process operates with constant returns to scale. The homogeneity of Ψ corresponds to the tautology that if we double the time during which a factory works, then the quantity of every flow element and the service of every fund will also double. The issue of returns to scale pertains, instead, to what happens if the fund elements are doubled. The point may be made still clearer.

A superficial inspection of any operating plant suffices to reveal that P consists of some Ricardian land, R, some capital equipment, K, some technical inventories, S, and a special fund, Γ, usually called "goods in process." The last term is definitely a misnomer: half-tanned hides or partly wired radio sets, for example, are not goods. Process-fund seems a more exact term because Γ is in effect a becoming frozen in its various phases. If a photograph of Γ would be projected part by part, as if it were a movie, we would witness the actual change of some input flows into product and waste flows. In spite of this varied composition of any plant, what a given plant can do depends on its buleprint alone, which in turn involves only L and K. And since what a plant can do is shown by the flow rate of its product, we have a first relation

$$(15) \qquad q^* = G^*(L, K).$$

A second relation expresses the fact that, given the plant, we require a certain labor power, H^*, if we want to obtain the flow rate q^*. Hence,

$$(16) \qquad H^* = H^*(L, K).$$

Should we man the plant with less labor power than H^*, the product flow rate would also become smaller than q^*. To account for this rather common situation, we need to put

$$(17) \qquad q = G(L, K, H) \leq q^*.$$

But the fact that this relation looks very familiar should not mislead us: as (17) is defined here, if $q<q^*$, q does not necessarily decrease (and ordinarily does not) when L and K are decreased while H is kept constant. Actually, the ratio q/q^* measures the percentage of capacity utilized if H is the labor power employed.[17]

The next relations are self-explanatory:

$$(18) \qquad S = S(L, K, H), \qquad \Gamma = \Gamma(L, K, H).$$

There remain the relations binding the other flow elements. The case of the maintenance flow, m, is simple: its size must depend on the amount of equipment to be maintained and the labor fund employed. In addition, by virtue of the conservation law of matter and energy, m must be equal to w_1, the flow rate of wear-and-tear waste—burned or discarded lubricating oil, broken saw bands, and the like. Hence,

$$(19) \qquad m = m(K, H), \qquad m = w_1.$$

The same conservation law applies to all other flows. For example, the wood contained in a piece of furniture together with the scrap and the sawdust must exactly account for the wood introduced into the production of that furniture. In the case of a factory system, this relationship yields

$$(20) \qquad qt = g(rt, it, w_2t),$$

where w_2 is the flow rate of waste arising from transformation alone. Since (20) must be true for any t, the function g must be homogeneous of the first degree. To this function we may indeed apply the old-time tautology that "doubling the inputs doubles the output." The basic error in some arguments about the returns to scale is to apply this tautology to (17) instead of (20). If L, K, and H are doubled, q does not necessarily double even if we double at the same time all flow inputs. The new factory may be more efficient or more wasteful in using the input materials, which leads us to put

$$(21) \qquad w_2 = w_2(L, K, H).$$

Relation (10) is thus decomposed into seven basic relations, listed from (17) to (21), which together constitute the general representation of a factory process.[18]

We should now note that the picture at which we have arrived is analogous to the inscription "60 watts" on an electric bulb. That is, it tells us what the factory can do, not what it has done, is doing, or, above all, will do. Like the inscription

[17] As an ordinal measure of the utilized capacity we may use H/H^*. On this point, see note 21, below.

[18] Obviously, this analytical description will have to be completed with additional relations if the particular factory process happens to involve other limitationalities.

on the bulb, relation (17), for example, is true regardless of whether the factory works or is idle. To show what the factory does, we need an additional coordinate, which, under its various aspects, has deeply preoccupied Marx, but which, perhaps for easily understood reasons, is not found in the analytical tool box of the neoclassical economist. The coordinate is the time, δ, during which the factory works daily. The amount of the daily production, Q, follows immediately from (17):

$$Q = \delta G(L, K, H), \tag{22}$$

a relation which vindicates Marx's dear tenet that labor time measures value even though it has no value itself.[19]

V

So much for grounded-in-actuality symbolism. Let me devote my closing remarks to some of the object lessons of this symbolism.

I have stressed the fact that in any elementary process every agent is idle over some definite periods that depend not on our choice or whim but on the state of the arts. I have also argued that we can nonetheless eliminate this kind of idleness completely and that there is only one way to achieve this: to arrange the elementary processes in a factory system. Because of this extraordinary property, the factory system deserves to be placed side by side with money as the two most fateful economic innovations for mankind. I say "economic" and not "technical" because the economy of time achieved by the factory system is independent of technology. Nothing prevents us from using the most primitive technique of cloth weaving in a factory system.

To be sure, there is a second kind of idleness, which depends entirely on our decision: it is the idleness of the factory itself if δ is shorter than a full day. In view of these two kinds of idleness, the economics of production reduces to two commandments: first, produce by the factory system and, second, let the factory operate around the clock.

The first commandment calls for two observations. Even though we can draw the blueprint of a factory for any elementary process whatsoever, not every such factory is economically advantageous. For example, we do not build transoceanic "Queens" by processes in line. The reason is that we can build a "Queen" more quickly than it is demanded in relation to time. The much extolled progress of the industrial revolution may not after all be due only to technological innovations. For these innovations as well as the increased specialization of labor could not have come about unless an increased demand had already induced most craft shops to introduce the system in line. There can be little doubt about it: the factory system was born in an artisan's workshop, not in a factory.

The second observation is that to operate an arrangement of elementary processes in line it is absolutely necessary that we have the freedom to start a process at any time of the day, of the week, and of the year. Unfortunately, we do not always have this freedom. Seasonal variations—which result from the position of our planet relative to our main source of free energy, the sun—prevent us from adapting the factory system to a series of important productive activities. The most important instance is husbandry. For the overwhelming majority of localities, there is a very short and definite period of the year during which a corn plant, for instance, can be grown in the open space from seed. This is why farmers have to work their fields in parallel; that is, in a system of production that yields practically no economy of time. The global analytical representation of that system is (8), not (9). The upshot is that the open-air factories, about which socialist writers in particular have been continuously raving, will remain a utopian dream as long as we are unable to alter the orbit of the earth.

The association between agriculture and the idleness of all agents involved is by now a commonplace. Still, not much is known or even suspected about the importance of the related loss. Two simple illustrations may bring out this point. For the first, let us consider one of the exceptional localities—such as the Island of Bali—where, because of an almost constant climate throughout the year, rice could be grown in an open-air factory. In this case, every day the same number of hands would move over the fields with the same funds of plows, buffaloes, sickles, and flails to plow, sow, harvest, and thresh. Every day the villagers would eat the rice sown that very day, as it were, and they would no longer have to bear the burden of the debts specific to agriculture. But most important of all, we would also discover that, without diminishing the old production at all, there would remain a substantial number of superfluous workers as well as a substantial stock of superfluous equipment—a palpable measure of the overcapitalization of farming in comparison with manufacturing. The second illustration pertains to the current technique by which chickens are raised in the United States. In fact, in this country there are no longer any chicken farms—even though the term continues to be used. Instead, there are chicken factories, with elemen-

[19] Marx, *Capital*, Vol. I, pp. 45, 588.

tary processes arranged in line. The "chicken war" of yesteryear would not have come about if the difference between the old and the new techniques had not been so great as to exceed the shipping cost over the Atlantic plus the wage differential between this country and Europe.

But if not every production activity can be turned into a factory, we should at least try to render the idleness of the agents as small as possible in each particular case. In other words, to bring even a whole economy as near as possible to the functioning of a factory system should be the guiding thought of any planner at any level. In the activity of the countryside, the cottage industry propounded by the agrarians was one answer to this idea. In Romania (so I was told) tractors and drivers shuttle between the plain regions—where two crops are grown each year—and the hilly regions—where only one crop can be grown because of a shorter vegetation period. The necessary funds of tractors and drivers are thus substantially reduced at the cost of some gasoline, oil, and spare parts flows. Less costly solutions would be obtained by mixing several crops within the same locality, the crops being chosen so as to minimize idleness (and hence capital cost). Formally, the problem boils down to splicing graphic patterns with a minimizing condition—a problem of a special type of combinatorial analysis which, I am sure, will prove highly rewarding.

The second commandment is particularly relevant for the underdeveloped economies. In a rich country, it makes perfect sense to operate every factory with one shift, even if the shift be of six or four hours only. In a rich country, there also is no need for night shifts, except whenever technology imposes around-the-clock production. Briefly, in a rich country leisure is a commodity which people may prefer to higher income. Things are different in almost every underdeveloped country where—as every government pronouncement urges—the order of the day is not only development but rapid development. In such countries, the regimen of the eight-hour working day and the reluctance to use night shifts are anachronistic factors that work against the avowed aims.[20] There may be many reasons why planners as well as our planning theory have overlooked the simplest and the most direct lever of economic development; namely, the length of the working day. But one possible reason is that this element of the problem has been left out of the neoclassical representation of a production process. The same omission—we should note—vitiates also the familiar comparisons of the capital-output and capital-labor ratios computed from current statistical data. Since the theoretical apparatus ignores the working time, δ, the most sophisticated statistical agencies, too, have felt no need to include it in their usual collections. Thus, we are unable to obtain valid statistical estimates of K/q^* and K/H^*, the basic technical and theoretical elements.[21]

Another omission of the neoclassical representation is that, as a rule, only the funds (variously defined) are included in the production function. The fact that after a factory is built, production cannot go on unless the input flow factors are forthcoming, has thus been pushed away from the focus of attention. None other than an authority such as A. C. Pigou preached that "in a stationary state factors of production are stocks, unchanging in amount, out of which emerges a continuous flow, also unchanging in amount, of real income."[22] The omission of the input flow factors is not unrelated to the present race of all underdeveloped countries to build one factory after another without a thorough examination of the availability of the necessary flow inputs. I am confident that if the prospective economic plan of every country were realized by miracle overnight, we would discover that we have long since been planning for a world with an immense excess capacity of industrial production.

The thoughts I shared with you here may seem simple. Perhaps they are simple, once we have untangled the imbroglio hatched by blind symbolism. The economics of production, its elementary nature notwithstanding, is not a domain where one runs no risk of committing some respectable errors. In fact, the history of every science, including that of economics, teaches us that the elementary is the hotbed of the errors that count most.

[20] I may hasten to admit that (22) is only a first approximation formula: a factory working with one shift of ten hours will not produce 25 percent more than with a shift of eight hours. To take better account of facts, we should replace δ by a function of the number of shifts and the number of working hours of each shift. But this amendment does not affect in the least the validity of the statements just made.

[21] The difficulty is especially serious in the case of comparisons between two different industries. Even if we know that each industry has always used its full capacity, i.e., has worked with the corresponding H^*, the values of capital-labor ratios derived from the usual statistical data are neither comparable nor strictly relevant—unless we also know that both industries employed the same number of shifts. In fact, the Census of Manufactures provides no information on the number of shifts and on the percentage of utilized capacity.

[22] Pigou, *op. cit.*, p. 19.